CORRECTION OF A TYPE OF DISTORTION SEEN IN BITEMARK PHOTOGRAPHY: A NOVEL ALGORITHM WITH A FORMAL PROOF

John D. Melville, MS, MD
David R. Senn, DDS

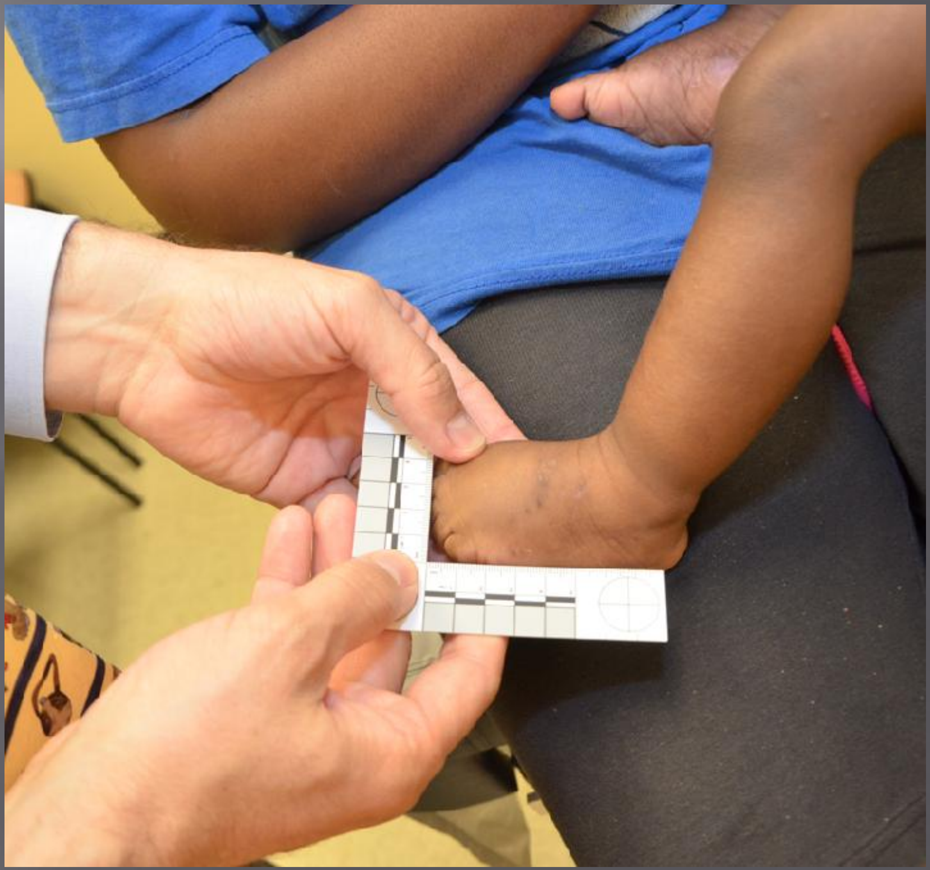
Disclosure

- Dr. Melville and Business Casual Software sell a commercial product that implements the algorithm described herein.


No.2 ©

Type 1 Distortion

- The scale and the portion of the injury being imaged are co-planar but the camera sensor (or film) is not parallel to that plane

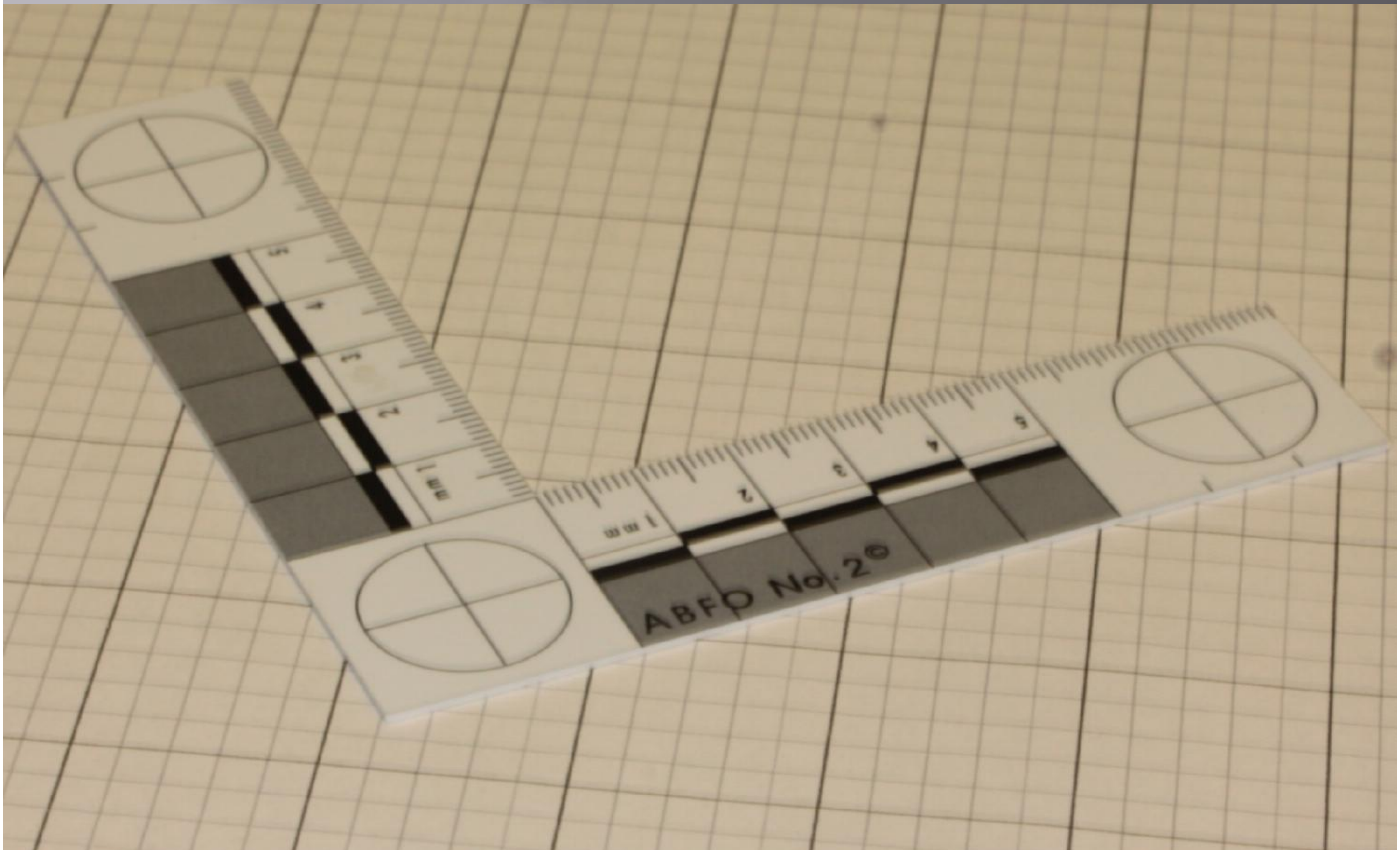


You can use a device to ensure that sensor and scale are in parallel planes

But, we do not always take our own photographs

from this

to this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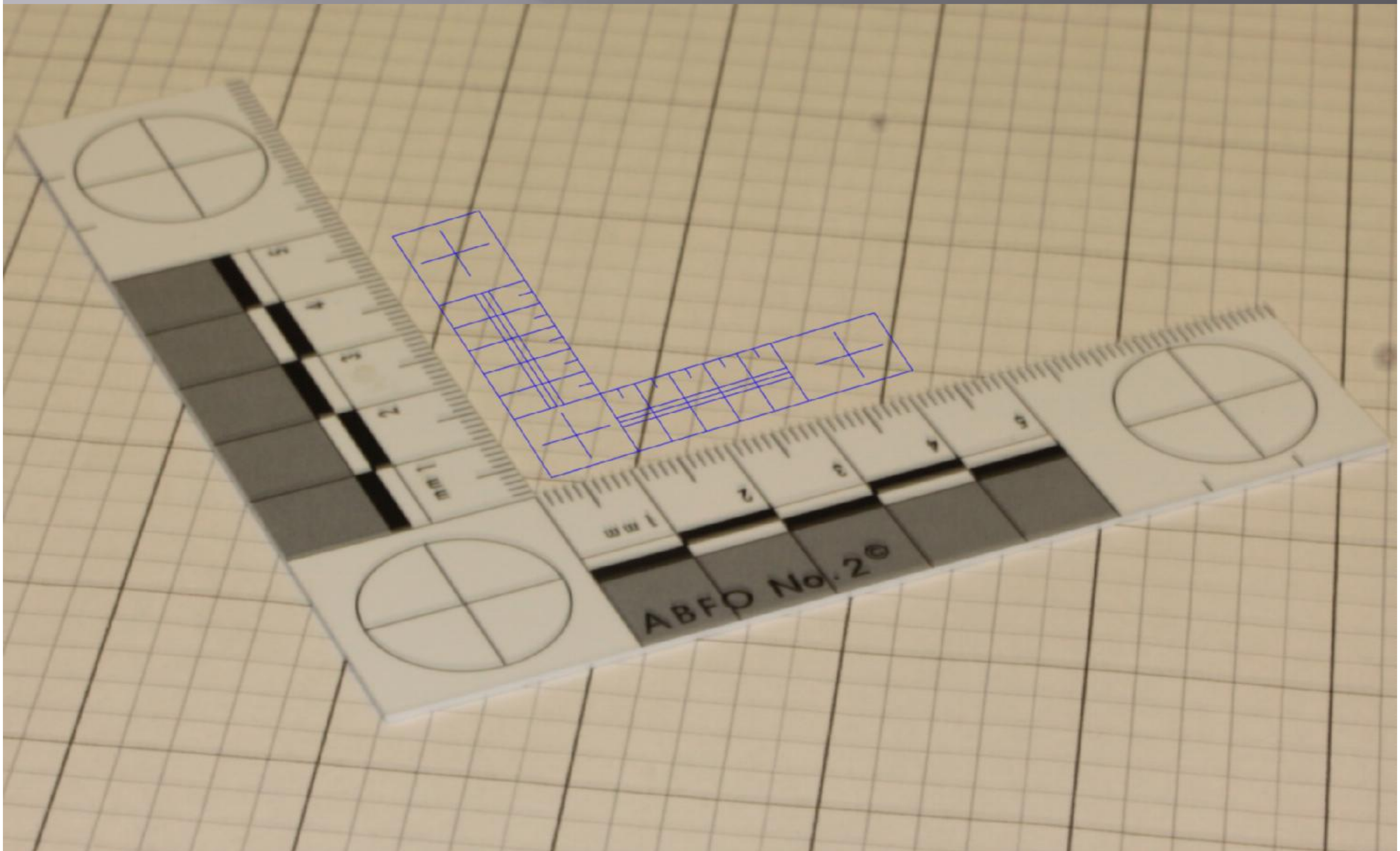
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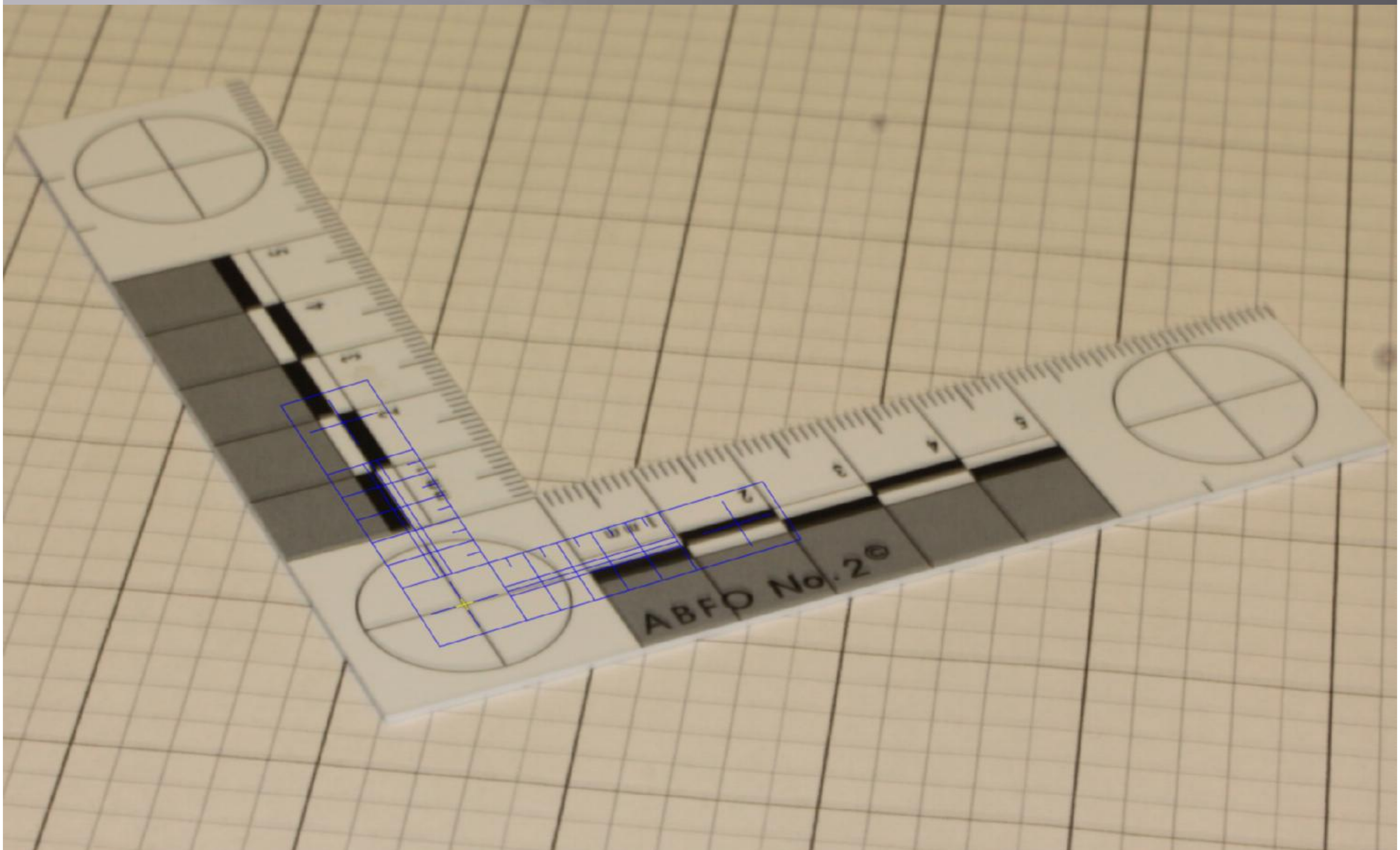
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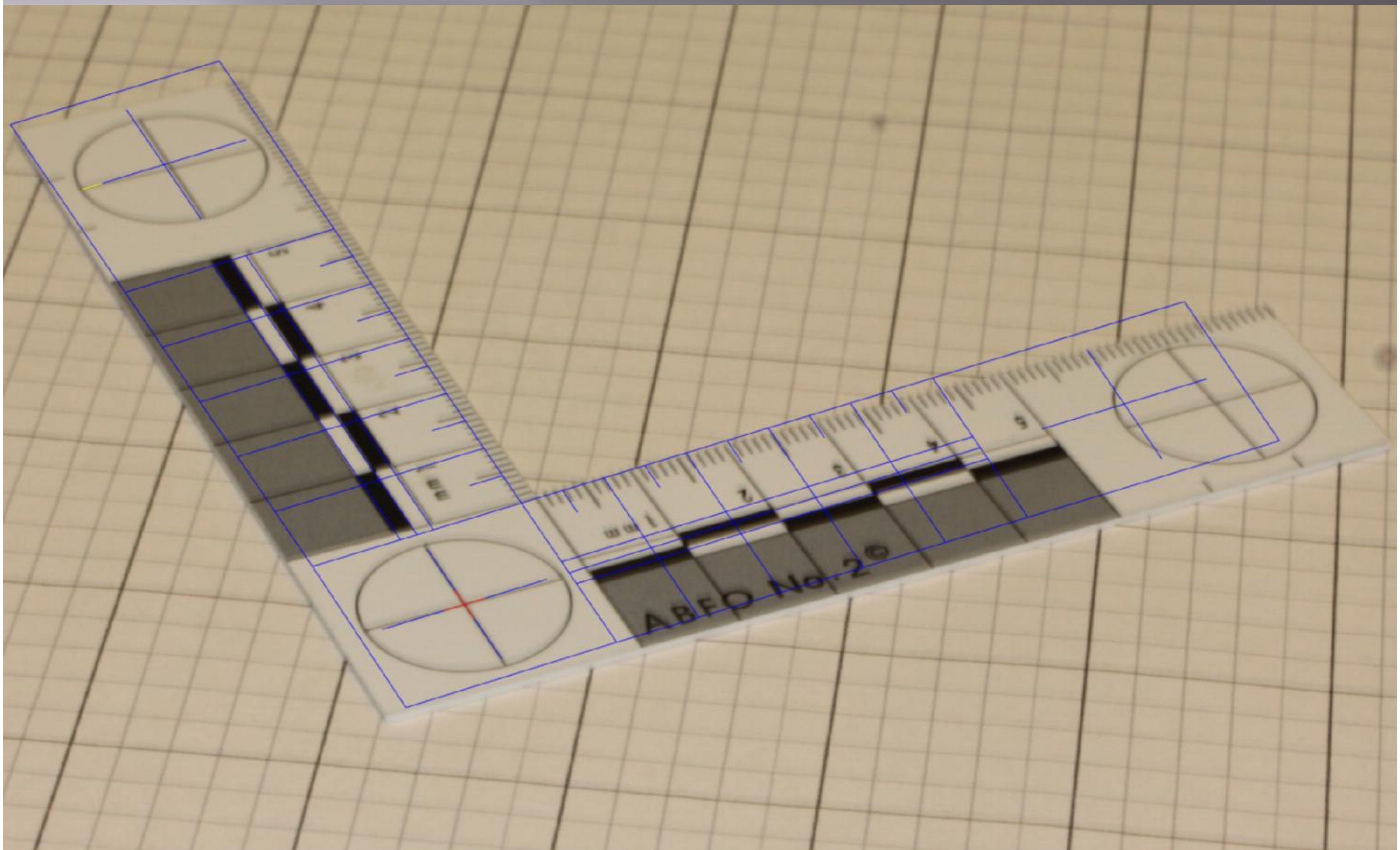
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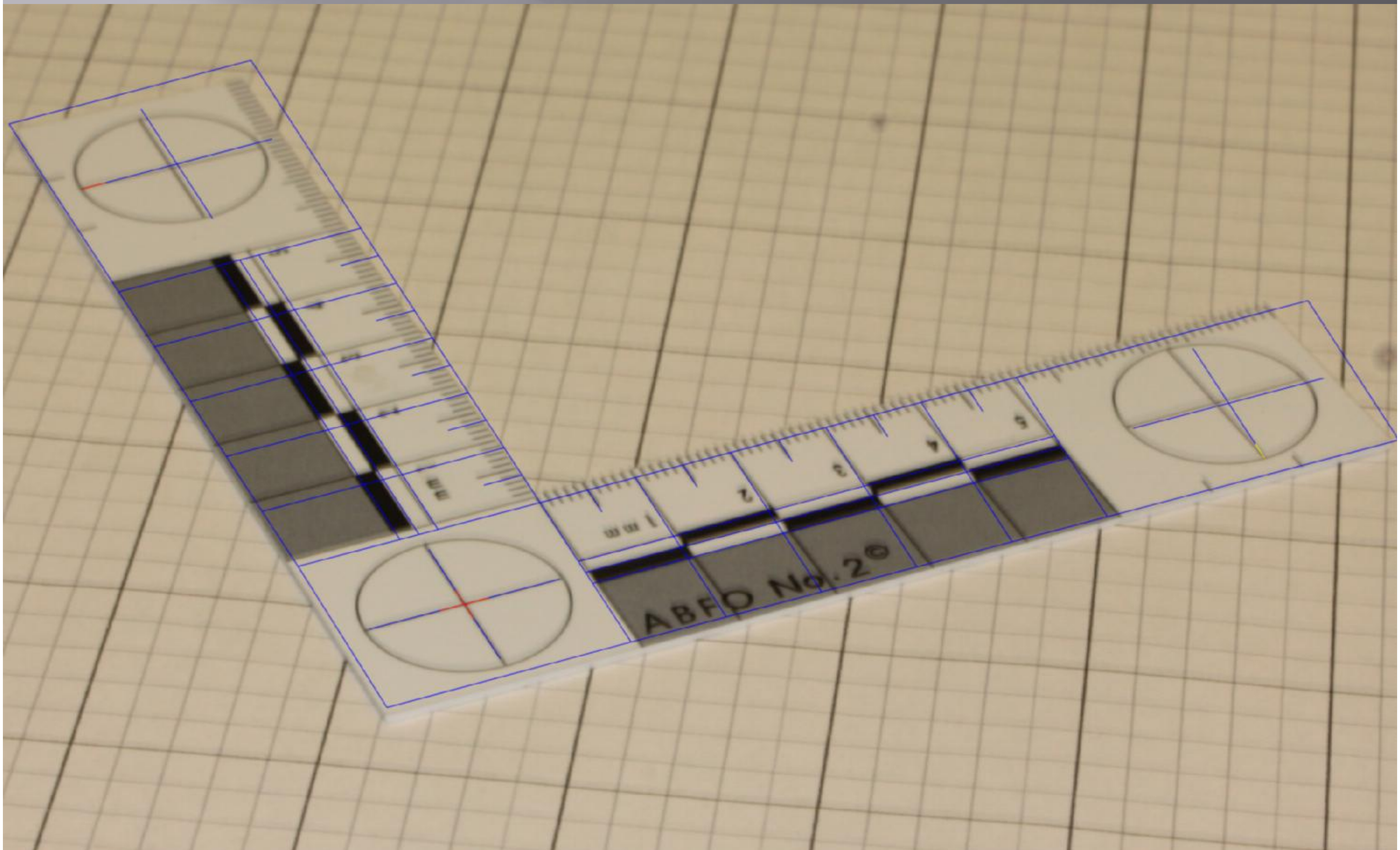
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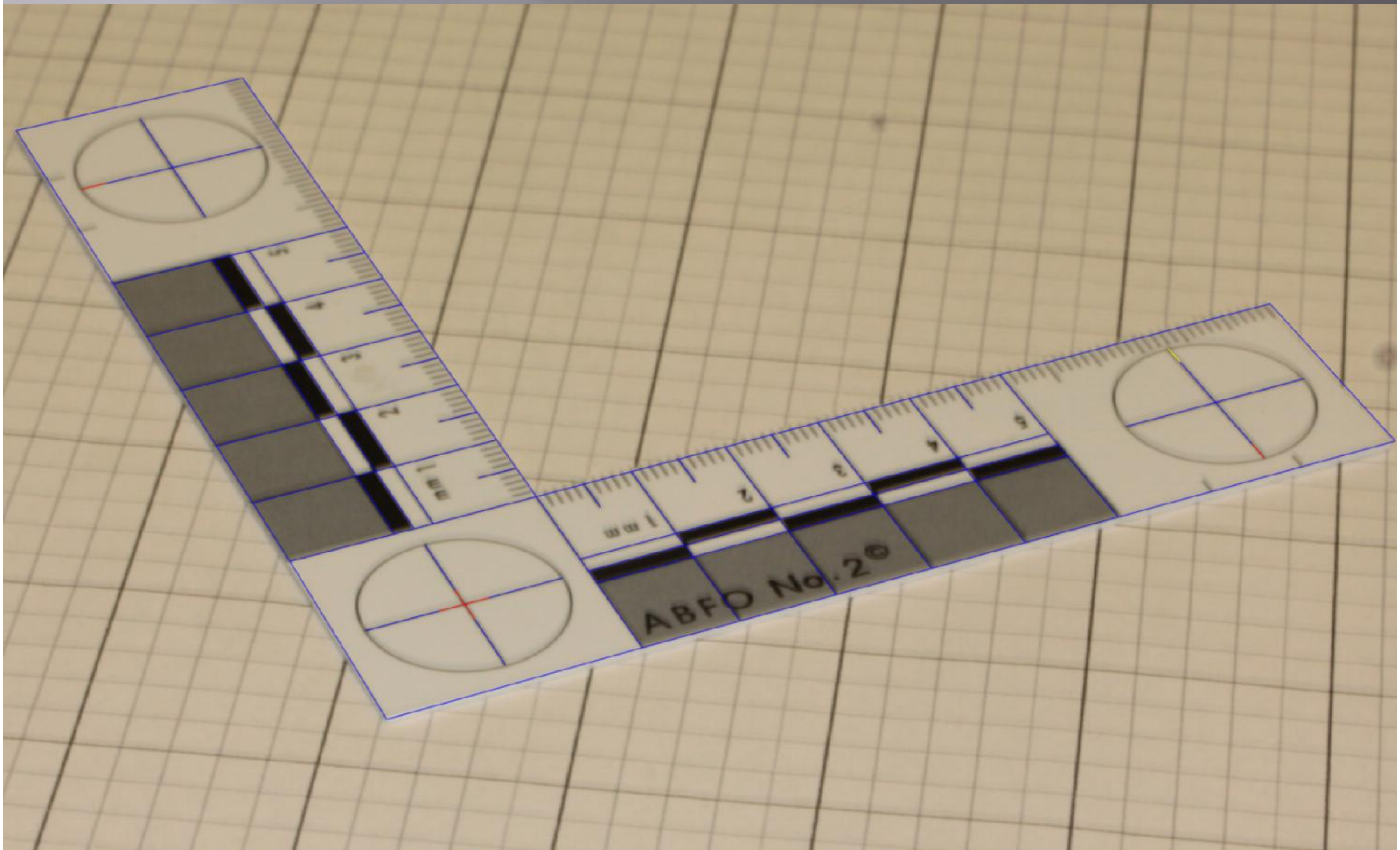
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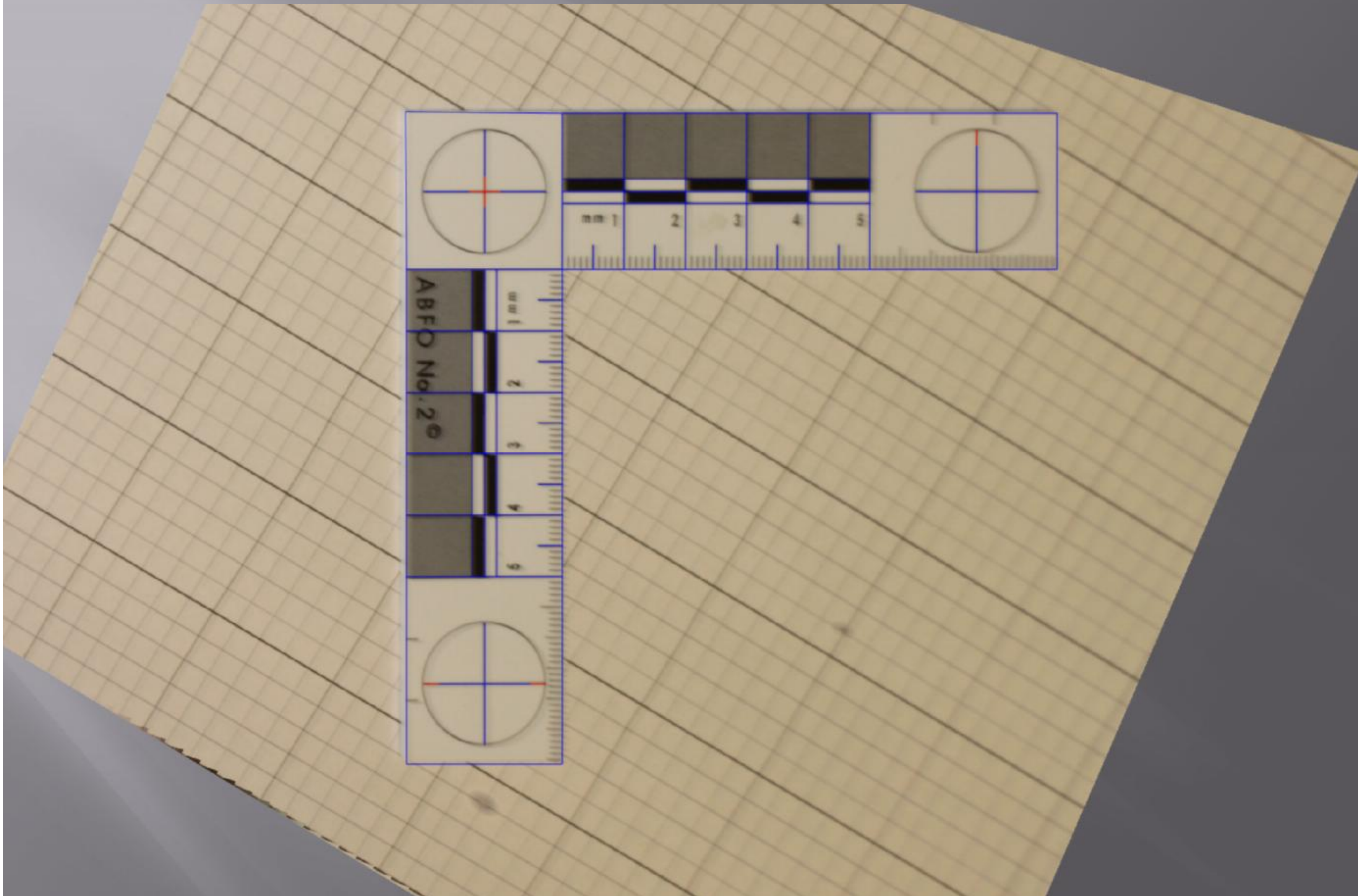
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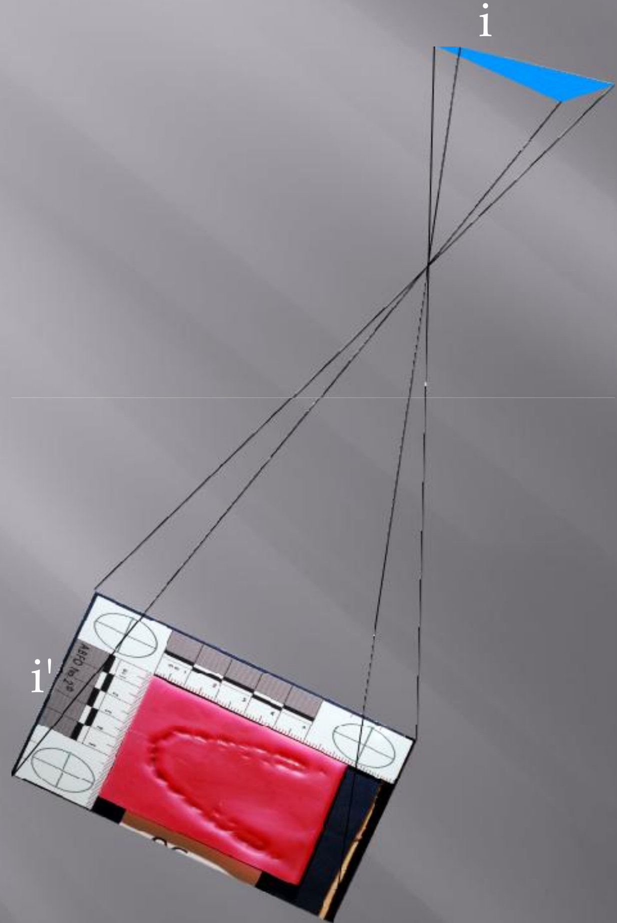
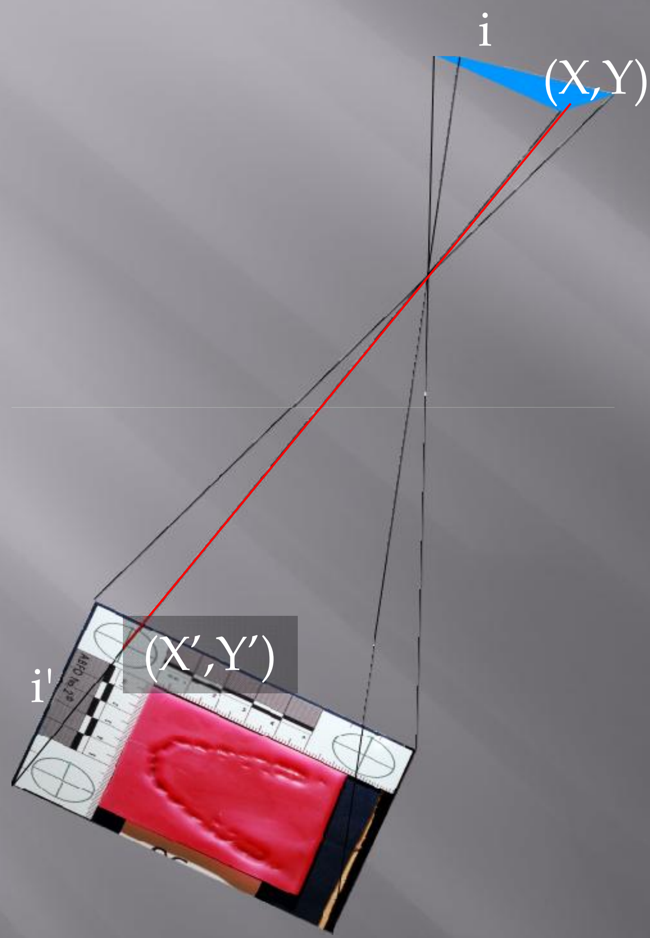
Proof of Correctness



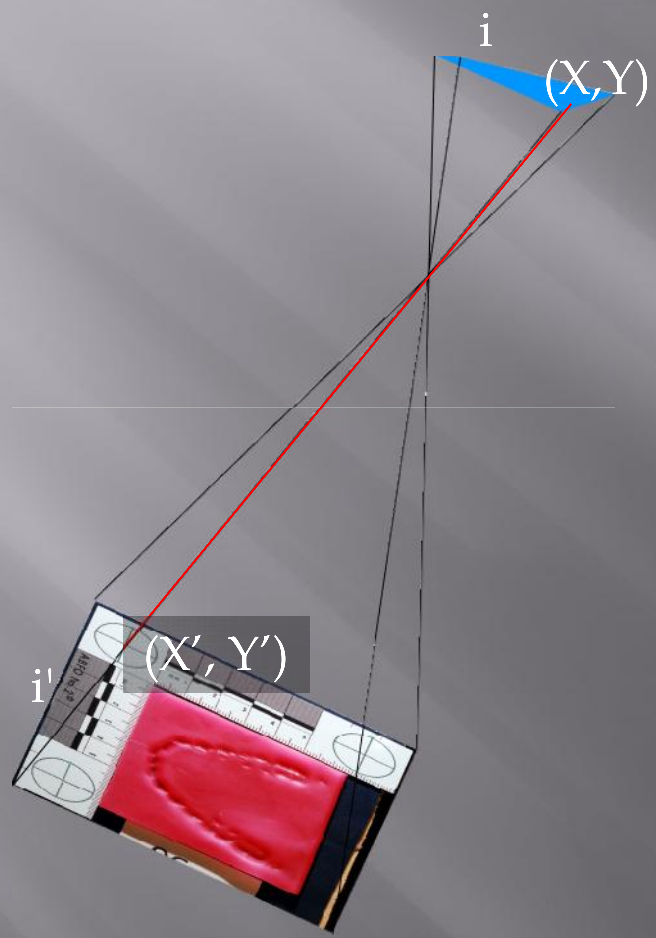
- Model the lens system as a pinhole camera.
- A camera maps plane i onto plane i’ using a lens.

Establishing Correspondance



- Placing a ruler point over the image maps point (X,Y) → (X′, Y′)

Defining the Transform



The 8 parameter transform between perspective planes states:

$$X' = \frac{f_1 X + g_1 Y + h_1}{f_0 X + g_0 Y + 1}$$

$$Y' = \frac{f_2 X + g_2 Y + h_2}{f_0 X + g_0 Y + 1}$$

Compute the 8 Parameters

Eight Parameter Transform Between Perspective Planes

$$X' = \frac{f_1X + g_1Y + h_1}{f_0X + g_0Y + 1} \qquad Y' = \frac{f_2X + g_2Y + h_2}{f_0X + g_0Y + 1}$$

Compute the 8 Parameters

Multiply both sides by $f_0X + g_0Y + 1$

$X'(f_0X + g_0Y + 1) = f_1X + g_1Y + h_1$

$Y'(f_0X + g_0Y + 1) = f_2X + g_2Y + h2$

Compute the 8 Parameters

Polynomial multiplication on left

$f_0 X' X + g_0 X' Y + X' = f_1 X + g_1 Y + h_1$ $\quad$ $f_0 Y' X + g_0 Y' Y + Y' = f_2 X + g_2 Y + h_2$

Compute the 8 Parameters

Collect Terms

$-f_0X'X + f_1X - g_0X'Y + g_1Y + h_1 = X'$

$-f_0Y'X + f_2X - g_0Y'Y + g_2Y + h_2 = Y'$

Compute the 8 Parameters

Add Zero Terms

$$-f_0X'X + f_1X + 0f_2 - g_0X'Y + g_1Y + 0g_2 + h_1 + 0h_2 = X'$$

$$-f_0Y'X + 0f_1 + f_2X - g_0Y'Y + 0g_1 + g_2Y + 0h_1 + h_2 = Y'$$

Compute the 8 Parameters

$$\begin{bmatrix} -X_1'X_1 & X_1 & 0 & -X_1'Y_1 & Y_1 & 0 & 1 & 0 \\ -Y_1'X_1 & 0 & X_1 & -Y_1'Y_1 & 0 & Y_1 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ g_0 \\ g_1 \\ g_2 \\ h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} X_1' \\ Y_1' \end{bmatrix}$$

Compute the 8 Parameters

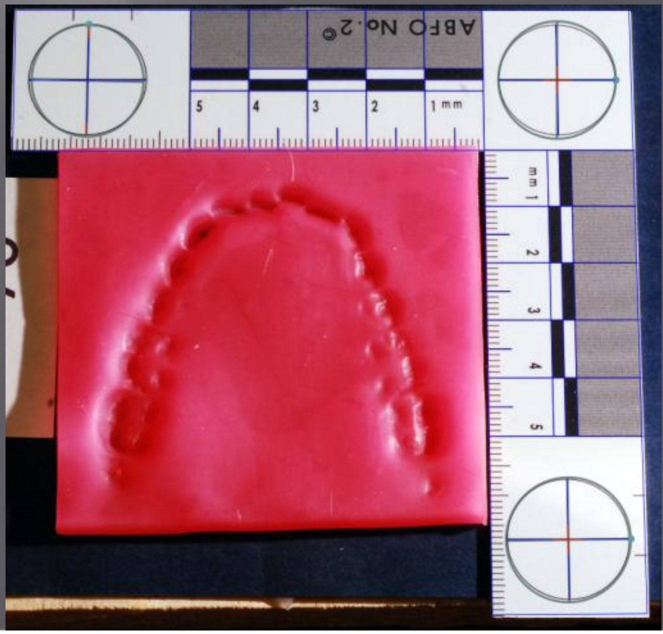
$$\begin{bmatrix} -X_1'X_1 & X_1 & 0 & -X_1'Y_1 & Y_1 & 0 & 1 & 0 \\ -Y_1'X_1 & 0 & X_1 & -Y_1'Y_1 & 0 & Y_1 & 0 & 1 \\ -X_2'X_2 & X_2 & 0 & -X_2'Y_2 & Y_2 & 0 & 1 & 0 \\ -Y_2'X_2 & 0 & X_2 & -Y_2'Y_2 & 0 & Y_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ g_0 \\ g_1 \\ g_2 \\ h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} X_1' \\ Y_1' \\ X_2' \\ Y_2' \end{bmatrix}$$

Compute the 8 Parameters

$$
\begin{bmatrix}
-X_1'X_1 & X_1 & 0 & -X_1'Y_1 & Y_1 & 0 & 1 & 0 \\
-Y_1'X_1 & 0 & X_1 & -Y_1'Y_1 & 0 & Y_1 & 0 & 1 \\
-X_2'X_2 & X_2 & 0 & -X_2'Y_2 & Y_2 & 0 & 1 & 0 \\
-Y_2'X_2 & 0 & X_2 & -Y_2'Y_2 & 0 & Y_2 & 0 & 1 \\
-X_3'X_3 & X_3 & 0 & -X_3'Y_3 & Y_3 & 0 & 1 & 0 \\
-Y_3'X_3 & 0 & X_3 & -Y_3'Y_3 & 0 & Y_3 & 0 & 1
\end{bmatrix}
\begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ g_0 \\ g_1 \\ g_2 \\ h_1 \\ h_2 \end{bmatrix}
=
\begin{bmatrix} X_1' \\ Y_1' \\ X_2' \\ Y_2' \\ X_3' \\ Y_3' \end{bmatrix}
$$

Compute the 8 Parameters

$$
\begin{bmatrix}
-X_1'X_1 & X_1 & 0 & -X_1'Y_1 & Y_1 & 0 & 1 & 0 \\
-Y_1'X_1 & 0 & X_1 & -Y_1'Y_1 & 0 & Y_1 & 0 & 1 \\
-X_2'X_2 & X_2 & 0 & -X_2'Y_2 & Y_2 & 0 & 1 & 0 \\
-Y_2'X_2 & 0 & X_2 & -Y_2'Y_2 & 0 & Y_2 & 0 & 1 \\
-X_3'X_3 & X_3 & 0 & -X_3'Y_3 & Y_3 & 0 & 1 & 0 \\
-Y_3'X_3 & 0 & X_3 & -Y_3'Y_3 & 0 & Y_3 & 0 & 1 \\
-X_4'X_4 & X_4 & 0 & -X_4'Y_4 & Y_4 & 0 & 1 & 0 \\
-Y_4'X_4 & 0 & X_4 & -Y_4'Y_4 & 0 & Y_4 & 0 & 1
\end{bmatrix}
\begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ g_0 \\ g_1 \\ g_2 \\ h_1 \\ h_2 \end{bmatrix}
=
\begin{bmatrix} X_1' \\ Y_1' \\ X_2' \\ Y_2' \\ X_3' \\ Y_3' \\ X_4' \\ Y_4' \end{bmatrix}
$$



Compute the 8 Parameters

$$
\begin{bmatrix}
-X_1'X_1 & X_1 & 0 & -X_1'Y_1 & Y_1 & 0 & 1 & 0 \\
-Y_1'X_1 & 0 & X_1 & -Y_1'Y_1 & 0 & Y_1 & 0 & 1 \\
-X_2'X_2 & X_2 & 0 & -X_2'Y_2 & Y_2 & 0 & 1 & 0 \\
-Y_2'X_2 & 0 & X_2 & -Y_2'Y_2 & 0 & Y_2 & 0 & 1 \\
-X_3'X_3 & X_3 & 0 & -X_3'Y_3 & Y_3 & 0 & 1 & 0 \\
-Y_3'X_3 & 0 & X_3 & -Y_3'Y_3 & 0 & Y_3 & 0 & 1 \\
-X_4'X_4 & X_4 & 0 & -X_4'Y_4 & Y_4 & 0 & 1 & 0 \\
-Y_4'X_4 & 0 & X_4 & -Y_4'Y_4 & 0 & Y_4 & 0 & 1 \\
 & & & . & & & & \\
 & & & . & & & & \\
 & & & . & & & & \\
-X_n'X_n & X_n & 0 & -X_n'Y_n & Y_n & 0 & 1 & 0 \\
-Y_n'X_n & 0 & X_n & -Y_n'Y_n & 0 & Y_n & 0 & 1
\end{bmatrix}
\begin{bmatrix}
f_0 \\ f_1 \\ f_2 \\ g_0 \\ g_1 \\ g_2 \\ h_1 \\ h_2
\end{bmatrix}
=
\begin{bmatrix}
X_1' \\ Y_1' \\ X_2' \\ Y_2' \\ X_3' \\ Y_3' \\ X_4' \\ Y_4' \\ . \\ . \\ . \\ X_n' \\ Y_n'
\end{bmatrix}
$$

Using the Parameters

Begin, again, with the 8 parameter transform between perspective planes.

$$X' = \frac{f_1 X + g_1 Y + h_1}{f_0 X + g_0 Y + 1}$$

$$Y' = \frac{f_2 X + g_2 Y + h_2}{f_0 X + g_0 Y + 1}$$

f's, g's, and h's are now known parameters of the transform

Using the Parameters

Expressed as a vector equation

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} \dfrac{f_1X + g_1Y + h_1}{f_0X + g_0Y + 1} \\ \\ \dfrac{f_2X + g_2Y + h_2}{f_0X + g_0Y + 1} \end{bmatrix}$$

Using the Parameters

Convert to heterogeneous coordinates.

$$\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \begin{bmatrix} \dfrac{f_1X + g_1Y + h_1}{f_0X + g_0Y + 1} \\ \dfrac{f_2X + g_2Y + h_2}{f_0X + g_0Y + 1} \\ 1 \end{bmatrix}$$

Using the Parameters

Multiply both sides by $f_0X + g_0Y + 1$.

$$\begin{bmatrix} a X' \\ a Y' \\ a \end{bmatrix} = \begin{bmatrix} f_1X + g_1Y + h_1 \\ f_2X + g_2Y + h_2 \\ f_0X + g_0Y + 1 \end{bmatrix}$$

Where $a = f_0X + g_0Y + 1$

Using the Parameters

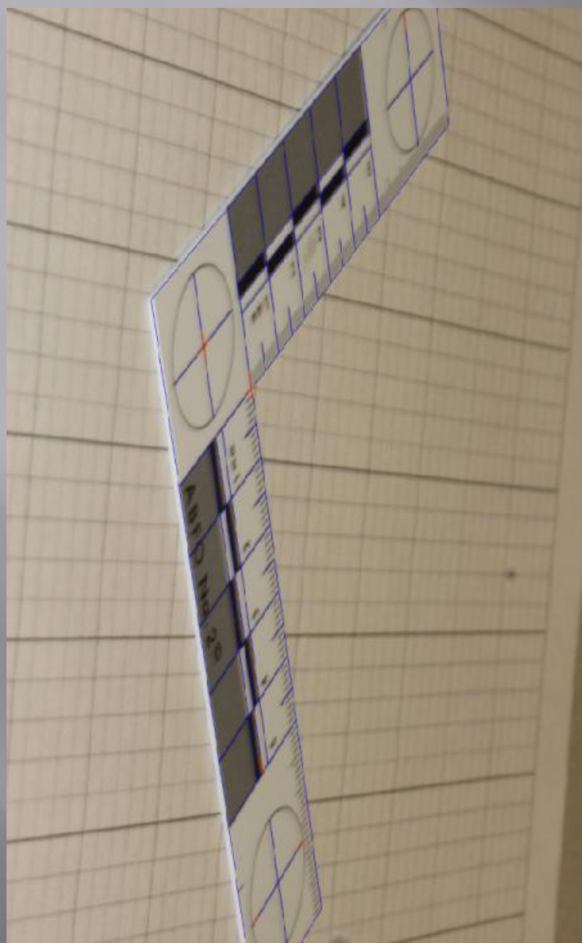
Factor the right hand side.

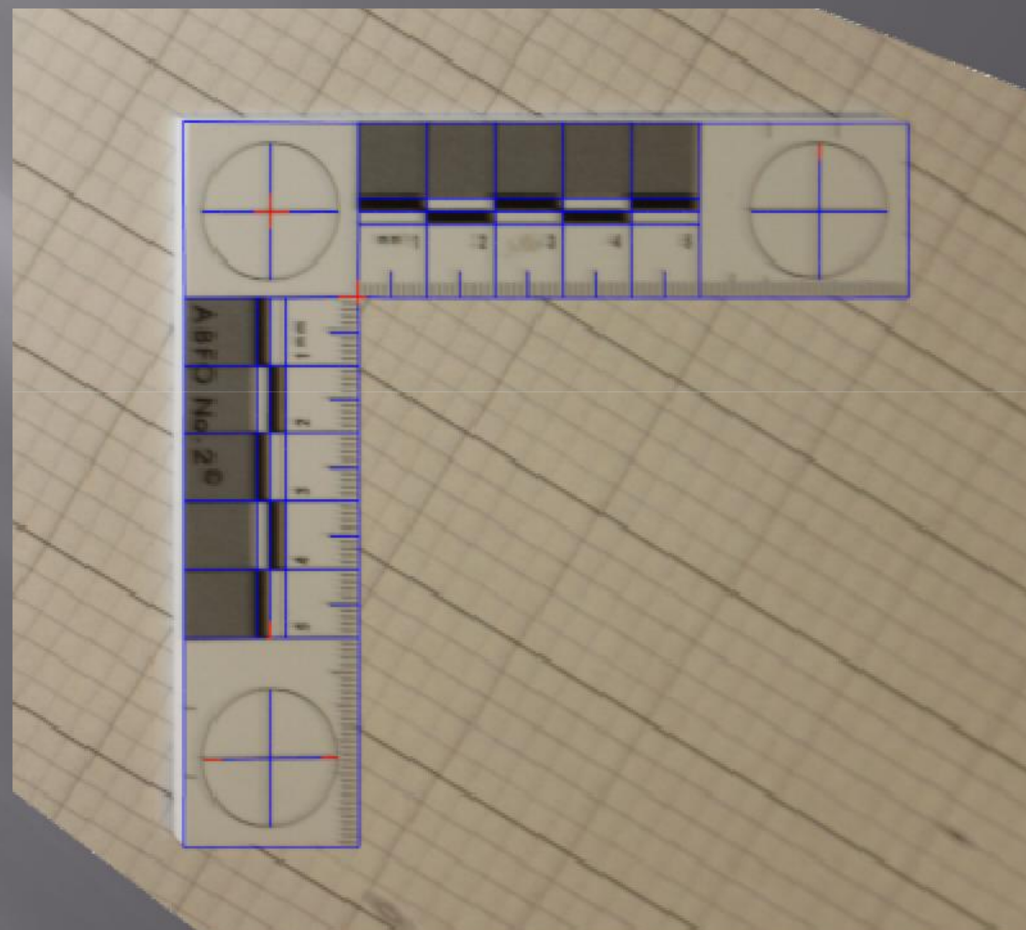
$$\begin{bmatrix} a X' \\ a Y' \\ a \end{bmatrix} = \begin{bmatrix} f_1 & g_1 & h_1 \\ f_2 & g_2 & h_2 \\ f_0 & g_0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Where $a = f_0 X + g_0 Y + 1$

Complex Angles

Extreme Angles





Torture Test

Oops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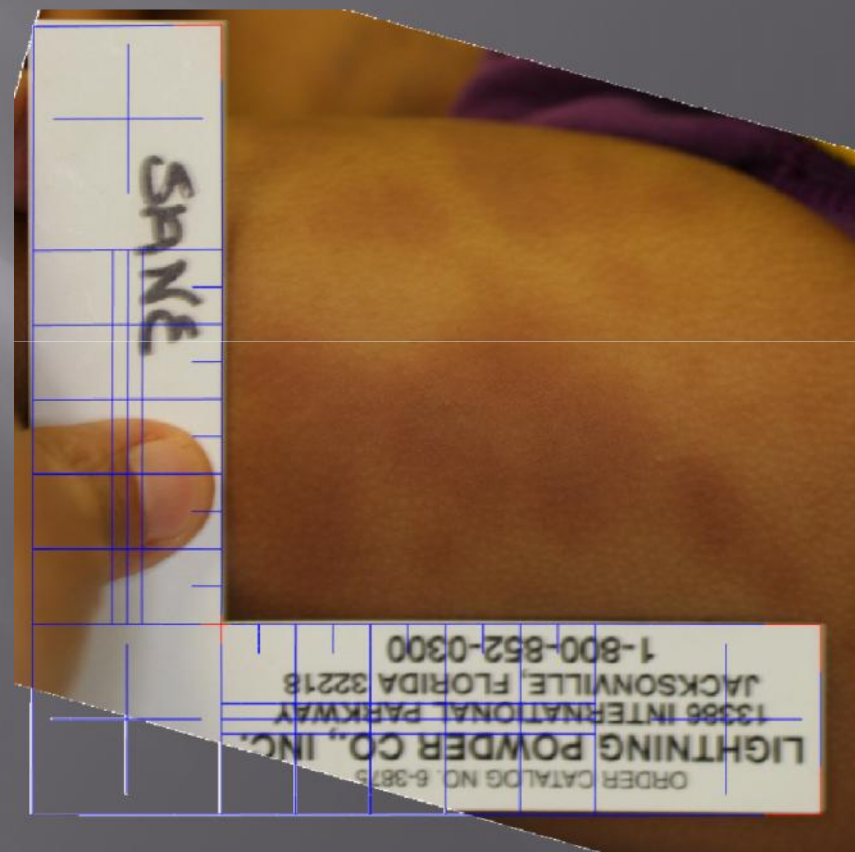
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Conclusion

PREVIOUSLY

- Time consuming
- 0^{o}-20^{o} (maybe 50^{o})
- Ad Hoc
- "Appears to Work"
- Frequently fails

THE NEW ALGORITHM

- 90 – 120 seconds
- 0^{o} - 80^{o}+
- Algorithm
- Proof of Correctness
- Robust to poor focus, complex angles, and partially occluded ruler.